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THE ROLE OF ABSORPTIVE CORRECTIONS IN A TRIPLE-REGGE ANALYSIS OF $pp \to \Delta^{++} + X$

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The size and nature of absorptive corrections in the triple-Regge region of inclusive reactions are discussed. A triple-Regge model using π, B, ρ and A_2 exchanges, both with and without absorption, is presented and compared with the data for $pp \to \Delta^{++}X$. Predictions for the x and p_T^2 dependence of the invariant cross section and the Δ^{++} density matrix elements are given. The effects of absorption in other π exchange dominated reactions are briefly discussed.

1. Introduction

For some time now it has been recognized that Regge pole exchange is able to provide a qualitative description of the systematics of two-body reactions [1]. However, in order to provide a quantitative description of such reactions it is necessary to allow for the presence of Regge cuts [2]. Virtually all of the phenomenological analyses to date have used some variation of the absorption model to generate these terms [3].

The need for Regge cuts in two-body reactions is made apparent through two types of effects seen in the data. The first of these is structure in differential cross sections. Regge cuts can be used to generate forward peaks in reactions such as $\gamma N \to \pi^{\pm} N$, $np \to pn$, and $\bar{p}p \to \bar{n}n$ where factorizing poles would lead to forward dips. Regge cuts have also been invoked to fill dips attributed to nonsense wrong signature zeros (NWSZ) in, for example, $\pi^- p \to \pi^0 n$ or $\pi N \to \omega N$. Alternatively, absorption has been used to generate dips in models which start with pole terms which do not possess NWSZ's [3].

The second effect of absorption is to alter the phases of the various helicity amplitudes. This can give rise to polarization effects which would be absent otherwise. Such phase modifications can also give rise to line-reversal breaking between reactions such as $K^+ n \to K^0 p$ and $K^- p \to \bar{K}^0 n$.

In the last few years much work has been done using the triple-Regge model for inclusive reactions [4]. It is clear by now that this model is capable of giving a qualitative description of the data in the triple-Regge region. Given the role played by absorptive corrections in two-body reactions, it is of interest to determine whether or not comparable effects are necessary to describe the triple-Regge region in the inclusive data. However, to date only the invariant cross section has been used for testing models. The smooth and relatively featureless structure of the data does not, as yet, indicate clearly the need for absorption. Furthermore, most analyses have been performed with reactions to which the triple-pomeron term can contribute. This greatly increases the number of terms in the models and simplifying assumptions are often made which makes it difficult to compare the results of different analyses [5, 6].

It is the purpose of this paper to point out that the reaction $pp \to \Delta^{++}X$ offers an indication that absorption may indeed be important in the triple-Regge region. This reaction is dominated by pion exchange and, due to the proximity of the pion pole to the physical region, it is possible to give a good estimate of the relevant triple-Regge coupling. The resulting prediction for the cross section is approximately a factor of two too large. This discrepancy may be removed by modifying the simple pole model by absorption.

The plan of the paper is as follows. In sect. 2 a formalism for generating absorptive corrections is presented and compared with the results of other authors [7–10]. In sect. 3 a model for the reaction $pp \to \Delta^{++}X$ is developed and compared with the data. In sect. 4 some features of absorbed π exchange are discussed while sect. 5 is reserved for a summary and conclusions. Some remarks concerning the effects of absorption in other reactions are also given in sect. 4. Certain technical aspects of the absorption calculation are discussed in an appendix.

2. Absorption in the triple-Regge model

We begin this section with a brief summary of the variables used to describe inclusive reactions and some of the relations between them. Consider the reaction

$$a + b \to c + X . \tag{1}$$

In terms of the 4-vectors of particles a, b, and c the usual Mandelstam variables are defined as

$$s = (P_a + P_b)^2 , \qquad t = (P_a - P_c)^2 , \qquad u = (P_b - P_c)^2 , \tag{2}$$

and the square of the missing mass is given by

$$M^2 = (P_a + P_b - P_c)^2 . \tag{3}$$

These four variables are related by

$$s + t + u = m_a^2 + m_b^2 + m_c^2 + M^2 , \tag{4}$$

so that only three, usually chosen as s, t and M^2, are independent.

It is also useful to work with the transverse and longitudinal momentum components, $p_\perp$ and $p_\parallel$, of particle c. The dependence on the longitudinal momentum (in the c.m. frame) is usually parametrized in terms of x where

$$x = p_\parallel / p_{\parallel \max} , \tag{5}$$

and $p_{\parallel \max} = \lambda(s, m_c^2, M_{\min}^2)^{1/2}/2\sqrt{s}$. Here $M_{\min}^2$ is the minimum physically allowed value of M^2 and the Kibble function $\lambda(s_a, s_b, s_c)$ is given by

$$\lambda(s_a, s_b, s_c) = [s_a^2 + s_b^2 + s_c^2 - 2s_a s_b - 2s_b s_c - 2s_a s_c] .$$

It is often necessary to relate values of x and $p_\perp^2$ to the corresponding values of M^2 and t. In the region of $x >> 0$ and $p_\perp^2$ small some useful relations are

$$M^2 \approx s(1 - x) - 2p_\perp^2/x + M_{\min}^2 x + m_c^2(1 + x - 2/x) , \tag{6}$$

$$t \approx (1 - x)(m_a^2 - m_c^2/x) - p_\perp^2/x . \tag{7}$$

Also, another variable, t', is often encountered where $t' = t - t_{\min} \approx -p_\perp^2/x$.

Reaction (1) is described in terms of amplitudes which are obtained by taking the discontinuity with respect to M^2 of an appropriate forward scattering amplitude for $ab\bar{c} \to ab\bar{c}$. This discontinuity will be denoted by $Dg_{a'b'c',abc}$ where the subscripts denote the final (primed) and initial (unprimed) particle helicities. This amplitude depends on only three variables which can be chosen, for example, as (s, t, M^2) or $(s, p_\perp^2, x)$.

As in two-body reactions, the absorptive corrections are attributed to rescattering effects. The effects of absorption on the input pole term, denoted by $Dg^{R}_{a'b'c',abc}$, are most easily seen by first transforming the pole term into impact parameter space. The net effect of the absorption is to reduce the amplitude at small-impact parameter. For the inclusive reaction, this transformation into impact parameter space is somewhat more complicated than for two-body reactions since the inclusive cross section may be thought of as being constructed from a sum of squares of amplitudes for "quasi two-body" reactions. Thus, two impact parameter transformations are required for the discontinuity $Dg^{R}_{a'b'c',abc}$.

The impact parameter representation can be written down simply by first defining the two-dimensional impact parameter, $\boldsymbol{b}$, to be the variable conjugate to the two-dimensional transverse momentum, $\boldsymbol{p}_\perp$, of particle c. This definition has proved to be useful both for studying large $p_\perp$ phenomena in two-body reactions [11] and for relating inclusive and exclusive $p_\perp$ distributions [12].

In order to generate the absorptive corrections the impact parameter transforma-

tion for the non-forward discontinuity must be specified. Using the above definition of impact parameter, this is easily written as

$$Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{p}_\perp,\boldsymbol{p}'_\perp)=\int\frac{\mathrm{d}^2b}{2\pi}\int\frac{\mathrm{d}^2b'}{2\pi}\,\mathrm{e}^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}\,\mathrm{e}^{-i\boldsymbol{b}'\cdot\boldsymbol{p}'_\perp}\,Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{b},\boldsymbol{b}')\,. \tag{8}$$

The inverse of this expression yields

$$Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{b},\boldsymbol{b}')=\int\frac{\mathrm{d}^2p_\perp}{2\pi}\int\frac{\mathrm{d}^2p'_\perp}{2\pi}\,\mathrm{e}^{-i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}\,\mathrm{e}^{i\boldsymbol{b}'\cdot\boldsymbol{p}'_\perp}\,Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{p}_\perp,\boldsymbol{p}'_\perp)\,. \tag{9}$$

For notational convenience only the $\boldsymbol{p}_\perp$ or $\boldsymbol{b}$ dependence is shown in the above expressions.

The absorption model is now obtained from eq. (8) by inserting the absorption profiles $S_{\mathrm{eff}}(\boldsymbol{b})$ and $S^*_{\mathrm{eff}}(\boldsymbol{b}')$ where $S_{\mathrm{eff}}(\boldsymbol{b})$ is the impact parameter transform of the effective S-matrix element. The absorbed amplitude for the inclusive process is then evaluated at $\boldsymbol{p}_\perp=\boldsymbol{p}'_\perp$, yielding

$$Dg_{a'b'c',abc}(\boldsymbol{p}_\perp)=\int\frac{\mathrm{d}^2b}{2\pi}\int\frac{\mathrm{d}^2b'}{2\pi}\,\mathrm{e}^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$

$$\times\,\mathrm{e}^{-i\boldsymbol{b}'\cdot\boldsymbol{p}_\perp}\,S_{\mathrm{eff}}(\boldsymbol{b})\,Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{b},\boldsymbol{b}')\,S^*_{\mathrm{eff}}(\boldsymbol{b}')\,. \tag{10}$$

Eq. (10) is not the most general expression for the absorbed triple-Regge model. In particular, it has been assumed that the absorption does not alter the helicities of the input pole term, i.e. the rescattering is dominantly non-flip. This assumption is reasonable to the extent that the pomeron helicity flip coupling is small.

Next, inserting eq. (9) into eq. (10) yields

$$Dg_{a'b'c',abc}(\boldsymbol{p}_\perp)=\int\frac{\mathrm{d}^2b}{2\pi}\int\frac{\mathrm{d}^2b'}{2\pi}\int\frac{\mathrm{d}^2k_\perp}{2\pi}\int\frac{\mathrm{d}^2k'_\perp}{2\pi}\,\mathrm{e}^{i\boldsymbol{b}\cdot(\boldsymbol{p}_\perp-\boldsymbol{k}_\perp)}\,\mathrm{e}^{-i\boldsymbol{b}'\cdot(\boldsymbol{p}_\perp-\boldsymbol{k}'_\perp)}$$

$$\times\,S_{\mathrm{eff}}(\boldsymbol{b})\,Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp)\,S^*_{\mathrm{eff}}(\boldsymbol{b}')\,. \tag{11}$$

The $\boldsymbol{b}$ and $\boldsymbol{b}'$ integrations are easily done yielding

$$Dg_{a'b'c',abc}(\boldsymbol{p}_\perp)=\int\frac{\mathrm{d}^2k_\perp}{2\pi}\int\frac{\mathrm{d}^2k'_\perp}{2\pi}\,S_{\mathrm{eff}}(\boldsymbol{p}_\perp-\boldsymbol{k}_\perp)\,Dg^{\mathrm{R}}_{a'b'c',abc}(\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp)\,S^*_{\mathrm{eff}}(\boldsymbol{p}_\perp-\boldsymbol{k}'_\perp)\,, \tag{12}$$

where

$$S_{\mathrm{eff}}(\boldsymbol{p}_\perp-\boldsymbol{k}_\perp)=\int\frac{\mathrm{d}^2b}{2\pi}\,\mathrm{e}^{i\boldsymbol{b}\cdot(\boldsymbol{p}_\perp-\boldsymbol{k}_\perp)}\,S_{\mathrm{eff}}(\boldsymbol{b})\,. \tag{13}$$

Eq. (12) is the general result which will be used for the following discussion. Further insight into the nature of the absorptive corrections can be gained by specifying

a form for $S_{\rm eff}(b)$. A simple Gaussian parametrization is chosen so that the integration in eq. (13) can be performed analytically. Let

$$S_{\rm eff}(b) = 1 - C\,{\rm e}^{-b^2/4a}\,. \tag{14}$$

The integration in eq. (13) can be performed using the expressions

$$\int_0^{2\pi} \frac{{\rm d}\phi}{2\pi}\,{\rm e}^{ibq\cos\phi} = J_0(bq)\,,$$

$$\int_0^{\infty} b\,{\rm d}b J_0(bq)\,{\rm e}^{-b^2/4a} = 2a\,{\rm e}^{-aq^2}\,,$$

$$\int \frac{{\rm d}^2 b}{2\pi}\,{\rm e}^{i\boldsymbol{b}\cdot\boldsymbol{q}} = 2\pi\delta^2(\boldsymbol{q})\,,$$

where ${\rm d}^2 b = b\,{\rm d}b\,{\rm d}\phi$. The result is

$$S_{\rm eff}(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp) = 2\pi\delta^2(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp) - 2aC\,{\rm e}^{-a(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp)^2}\,. \tag{15}$$

Therefore, eq. (12) takes the form

$$Dg_{a'b'c',\,abc}(\boldsymbol{p}_\perp) = \int \frac{{\rm d}^2 k_\perp}{2\pi}\int \frac{{\rm d}^2 k'_\perp}{2\pi}\,Dg^{\rm R}_{a'b'c',\,abc}(\boldsymbol{k}_\perp, \boldsymbol{k}'_\perp) \tag{16}$$

$$\times\,\{2\pi\delta^2(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp) - 2aC\,{\rm e}^{-a(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp)^2}\}\{2\pi\delta^2(\boldsymbol{p}_\perp - \boldsymbol{k}'_\perp) - 2a^*C^*\,{\rm e}^{-a^*(\boldsymbol{p}_\perp - \boldsymbol{k}'_\perp)^2}\}\,.$$

Eq. (16) for the absorbed amplitude involves four distinct terms. In the first term the product of the δ functions simply reproduces the pole term. For each of the cross terms only one integration is necessary, the other one being performed trivially using the single δ function. These two terms give rise to destructive interference with the pole term. Note that for the case $a' = a$, $b = b'$, $c' = c$ the sum of the cross terms is real, as is required, since $Dg_{abc,\,abc}$ contributes to the cross section. Finally, the fourth term involves a double integration. This term gives a positive contribution to the cross section as is necessary in order to preserve the positivity of $Dg_{abc,\,abc}$. If this term were absent, then strong absorption could yield a negative result for the cross section as a result of pole-cut interference.

The form of eq. (16) can be understood in another fashion, by analogy with two-body reactions. Denote the reggeon and reggeon-pomeron contributions to a typical two-body amplitude by R and $R \times P$. Then the absorbed amplitude is just

$$f \sim R + R \times P\,.$$

The cross section contains $|f|^2$ which is just

$$|f|^2 \sim |R|^2 + R^*(R \times P) + R(R \times P)^* + (R \times P)(R \times P)^*\,.$$

These four terms are analogous to the four terms appearing in eq. (16).

Other authors have presented expressions for absorptive corrections in inclusive reactions [7–10] and it is useful to compare the various results. Craigie and Kramer [7] arrived at a form similar to eq. (16) by first performing a partial wave decomposition of the inclusive amplitude and then converting the result into an impact parameter representation. However, the impact parameter utilized in that analysis was not defined as being the variable conjugate to the transverse momentum of particle c. Therefore, the integration variable used in [7] differs from that in eq. (16). Specifically, their integration variable is $\tau = p_\perp/x$. This results in a slight modification of the x dependence of the absorbed amplitude. However, following the discussion in refs. [11,12] it seems preferable to adopt the definition of impact parameter used here. From a phenomenological viewpoint this difference is not likely to be noticeable.

In ref. [8] a form similar to eq. (12) is derived using the AGK [13] cutting rules for reggeon diagrams. The effective rescattering term is expressed in an eikonal form which is then parametrized to include the effects of inelastic intermediate states. This amounts to selecting a specific form for $S_{\text{eff}}(b)$ in eq. (12). However, in the analysis in ref. [8] the relation $t' = -p_\perp^2$ is employed instead of $t' = -p_\perp^2/x$. Since the analysis is restricted to values of x near 1 this does not alter their conclusions.

In ref. [9] another derivation of the absorption model is presented utilizing essentially the same arguments presented here. However, the effective scattering term $S_{\text{eff}}(b)$ is attributed to rescattering of the outgoing particle c on the target particle b. Therefore, the rescattering is parametrized in terms of $s' = (P_c + P_b)^2 \approx xs$. Since the energy dependence of S_{eff} is weak this should have a negligible effect on the result.

In ref. [10] the absorption model is developed by studying the structure of certain hybrid diagrams in the context of the reggeon calculus. The Regge cuts which appear contain the residues of certain fixed poles in the reggeon-particle scattering amplitudes. These residues are identical to those appearing in two-body reactions. Replacing the fixed pole residue by an on mass shell intermediate state then yields the absorption model.

All of the models discussed above can be cast in a form similar to eq. (12). Furthermore, each model predicts that the absorption will be comparable to that in two-body reactions. There exists one model, however, which does differ formally from eq. (12). In ref. [14] Craigie et al. derive an expression using the funtional derivative technique of Abarbanel and Itzykson [15] to sum the set of all possible pomeron mediated rescatterings of particles a and b and particles c and b. This results in an eight-dimensional integral representation of the fully absorbed amplitude. The effects due to the more complicated structure of this model are currently being investigated [33].

In the next section a triple-Regge pole model is developed for the reaction $pp \to \Delta^{++}X$ and compared with the data. Then, absorptive corrections are calculated using eq. (16) and their effects are investigated.

3. $pp \to \Delta^{++}X$

Before developing the model for $pp \to \Delta^{++}X$ it is first necessary to give the expressions for the relevant observables in terms of the inclusive amplitudes. In each of the observables to be used in this analysis the target proton helicity is averaged over. Therefore, it is convenient to define a target helicity averaged amplitude by

$$Dg_{a',c';a,c} = \tfrac{1}{2}[Dg_{a',1/2,c';a,1/2,c} + Dg_{a',-1/2,c';a,-1/2,c}] \,. \tag{17}$$

Then, the invariant cross section, averaged over the azimuthal angle, is given by

$$\frac{E}{p_{\parallel\max}} \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}p_\perp^2} = \frac{1}{16\pi^2 s} \Sigma \,, \tag{18}$$

where

$$\Sigma = Dg_{1/2,3/2;1/2,3/2} + Dg_{-1/2,3/2;-1/2,3/2} + Dg_{1/2,1/2;1/2,1/2} + Dg_{-1/2,1/2;-1/2,1/2} \,.$$

The Δ density matrix elements are given by

$$\rho_{mm'} = \sum_{ab} Dg_{a,b,m';a,b,m} \Big/ \sum_{abc} Dg_{a,b,c;a,b,c} \,,$$

for the case of an unpolarized beam and target [16]. In this case the density matrix elements ρ_{33}, $\mathrm{Re}\rho_{31}$, and $\mathrm{Re}\rho_{3-1}$ can be deduced from the decay angular distribution. The following expressions can be obtained for these density matrix elements in terms of the amplitudes in eq. (17):

$$\begin{aligned}
\rho_{33} &= [Dg_{1/2,3/2;1/2,3/2} + Dg_{-1/2,3/2;-1/2,3/2}]/2\Sigma \,, \\
\mathrm{Re}\rho_{31} &= \mathrm{Re}\,[Dg_{1/2,3/2;1/2,1/2} + Dg_{-1/2,3/2;-1/2,1/2}]/2\Sigma \,, \\
\mathrm{Re}\rho_{3-1} &= \mathrm{Re}\,[Dg_{1/2,3/2;1/2,-1/2} + Dg_{-1/2,3/2;-1/2,-1/2}]/2\Sigma \,.
\end{aligned} \tag{19}$$

It is expected that at least four exchanges will contribute to $pp \to \Delta^{++}X$: π, B, ρ and A_2 *. Of these, the π exchange term dominates at small t. The form expected from elementary π exchange is well known [18]. The reggeized π exchange term is then written in such a way that it reduces to the elementary π result at the π pole.

* The effects due to the exchange of either the A_1 or its exchange degenerate partner (Z with $J^{PC} = 2^{--}$ [17]) will not be considered here.

The π exchange contribution to the invariant cross section then takes the form

$$\frac{E}{p_{\parallel\max}}\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}p_\perp^2} = \frac{G_{\pi N\Delta}^2}{4\pi}\frac{\alpha_\pi'^2}{4\pi}\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-1}\Gamma(-\alpha_\pi(t))^2\,\tfrac{1}{2}(1+\cos\alpha_\pi(t))$$

$$\times\,\sigma_{\mathrm{tot}}(\pi^-\mathrm{p})K(t)\,\mathrm{e}^{A_\pi(t-m_\pi^2)}\,, \tag{20}$$

where $K(t) = [(m+m_\Delta)^2 - t]^2[(m-m_\Delta)^2 - t]/6m_\Delta^2$. The π trajectory is given by $\alpha_\pi(t) = \alpha_\pi'(t - m_\pi^2)$ with $\alpha_\pi' = 0.7\ (\mathrm{GeV}/c)^{-2}$. The coupling constant appearing in (20) may be related to the Δ width by

$$\Gamma(\Delta\to \mathrm{N}\pi) = \frac{G_{\pi N\Delta}^2}{4\pi}\frac{k^3}{6m_\Delta^2}[(m+m_\Delta)^2 - m_\pi]\,, \tag{21}$$

where $k = \lambda(m_\Delta^2, m^2, m_\pi^2)^{1/2}/2m_\Delta$. A width of 110 MeV yields $G_{\pi N\Delta}^2/4\pi = 17.4\ \mathrm{GeV}^{-2}$.

The factor $\sigma_{\mathrm{tot}}(\pi^-\mathrm{p})$ is the total cross section for (virtual) $\pi^-\mathrm{p}$ scattering. Due to the proximity of the π pole to the physical region, only a small extrapolation is required. Therefore, using the physical $\pi^-\mathrm{p}$ total cross section should be a good approximation. A good description of the $\pi^-\mathrm{p}$ total cross-section data [19], in the energy region we will consider, is given by

$$\sigma_{\mathrm{tot}}(\pi^-\mathrm{p}) = (21 + 26/\sqrt{s})\ \mathrm{mb}\,. \tag{22}$$

Thus, the only unspecified parameter is A_π which is used to parametrize the pion form factor. A_π is well determined by the data for $\mathrm{d}\sigma/\mathrm{d}p_\perp^2$.

Having specified the π exchange contribution, it is possible to estimate the contribution for B exchange. Exchange degeneracy suggests that, apart from the signature factor, the B and π exchange terms should be equal. Furthermore, with $\alpha_B = \alpha_\pi$ there are no π-B interference terms since the two exchanges are then ninety degrees out of phase. However, several analyses for $\pi\mathrm{N}\to\mathrm{VN}$ [17] and $\pi\mathrm{N}\to\mathrm{V}\Delta$ [20] have shown that the B exchange contribution is approximately a factor of two larger than the exchange degeneracy estimate. On the other hand, the data for $\mathrm{pp}\to\Delta^{++}\mathrm{X}$ are not sensitive to the precise amount of B exchange, since the net B contribution includes a factor of $(1 - \cos\pi\alpha_B(t))$ which is near zero for small $p_\perp^2$. Therefore, in the following analysis the exchange degeneracy estimate for B exchange will be used. The resulting expression for the invariant cross section, including both π and B exchange, is

$$\frac{E}{p_{\parallel\max}}\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}p_\perp^2} = \frac{G_{\pi N\Delta}^2}{4\pi}\frac{\alpha_\pi'^2}{4\pi}\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-1}\Gamma(-\alpha_\pi(t))^2\,\sigma_{\mathrm{tot}}(\pi^-\mathrm{p})K(t)\mathrm{e}^{A_\pi(t-m_\pi^2)} \tag{23}$$

In order to calculate the density matrix elements (19), as well as to calculate the absorptive corrections, it is necessary to determine the helicity amplitude structure

Table 1
The vertex factors $V_{\lambda_\Delta \lambda_N}$ for π, B, ρ and A_2 exchange

λ_Δ	λ_N	π, B	ρ, A_2
$\frac{3}{2}$	$\frac{1}{2}$	$-(m_\Delta/x + m)/\sqrt{2x}$	$(m_\Delta + mx)/2\sqrt{2x}$
$\frac{3}{2}$	$-\frac{1}{2}$	$-1/x\sqrt{2x}$	$1/2\sqrt{2x}$
$\frac{1}{2}$	$\frac{1}{2}$	$[-t' + (m_\Delta + mx)(m^2x - m_\Delta^2/x - t')/m_\Delta]/\sqrt{6x}$	$-1/2\sqrt{6x}$
$\frac{1}{2}$	$-\frac{1}{2}$	$[-(m_\Delta/x + m) + (m^2x - m_\Delta^2/x - t')/m_\Delta]/\sqrt{6x}$	$(m_\Delta + mx)/2\sqrt{6x}$

for the π and B exchange terms. The result is *

$$Dg_{\lambda'_N,\lambda'_\Delta;\lambda_N,\lambda_\Delta} = G^2_{\pi N\Delta}\alpha'^2_\pi \left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)} \Gamma(-\alpha_\pi(t))^2 M^2 \sigma_{tot}(\pi^- p)$$

$$\times e^{A_\pi(t-m_\pi^2)} V_{\lambda'_\Delta\lambda'_N} V_{\lambda_\Delta\lambda_N} (p_\perp)^{|m|+|m'|}, \qquad (24)$$

where $m = \lambda_N - \lambda_\Delta$, $m' = \lambda'_N - \lambda'_\Delta$, and the vertex functions, $V_{\lambda_\Delta\lambda_N}$, are given in table 1.

From table 1 it can be seen that the vertex factors $V_{1/2\,1/2}$ and $V_{1/2\,-1/2}$ are t dependent and that they possess zeros in the physical region. In analyses of both $\gamma N \to \pi\Delta$ [21] and $\pi^+ p \to (\rho^0, \omega)\Delta^{++}$ [22] this additional t dependence was found to have little effect on the observables. Furthermore, the amplitude analyses in [20] found no indication of this extra structure. In all of the preceding analyses, it was found that a satisfactory description of the data could be obtained with the factors $V_{1/2\pm 1/2}$ evaluated at the π pole, i.e. $t' = m_\pi^2 - t_{min}$. This procedure will be employed in the present analysis as well.

Next, it is necessary to specify the structure of the ρ and A_2 exchange contributions. The helicity dependence of the $\rho N\Delta$ vertex is specified by using the Stodolsky-Sakurai model [23]. Exchange degeneracy can then be used to estimate the A_2 contribution. However, it is known that $\alpha_\rho - \alpha_{A_2} \sim 0.1$ [24] and, furthermore, the A_2 term is somewhat smaller than ρ–A_2 exchange degeneracy would suggest [17,20]. The trajectory splitting means that the ρ and A_2 exchanges are no longer ninety degrees out of phase and that, therefore, ρ–A_2 interference terms should be included in the calculation. However, such terms correspond, in the triple-Regge model, to off shell $\rho p \to A_2 p$ scattering which is mediated by odd G parity exchange (ω and A_2). Such contributions are suppressed by a factor of approximately $1/M$ relative to the $\rho\rho$ and A_2A_2 terms. Thus, the data are not sensitive to small amounts of trajectory splitting or exchange degeneracy breaking. This conclusion is further strengthened by the observation that the ρ and A_2 terms contribute only at moderate $p_\perp^2$ values, as will be

* For additional discussions concerning the vertex functions $V_{\lambda_\Delta\lambda_N}$, see refs. [20,21].

discussed shortly. Therefore, the exchange degeneracy relation for ρ and A_2 exchange will be used in the following analysis.

The general form for the $\rho + A_2$ contribution to a specific helicity amplitude will contain a sum of triple-Regge couplings including $G_{\rho\rho P}$, $G_{\rho\rho f}$ and $G_{\rho\rho\rho}$. The f-coupled pomeron hypothesis predicts that $G_{\rho\rho F} = G_{\rho\rho f}$ [25]. Furthermore, exchange degeneracy predicts that $G_{\rho\rho f}/G_{\rho\rho\rho} = \beta_{fNN}/\beta_{\rho NN}$ where β_{fNN} and $\beta_{\rho NN}$ are the non-flip reggeon-nucleon-nucleon couplings. This ratio is known to be about 4 from SU(3) $(d/f)_{++}$ ratios [26]. Therefore we take

$$G_{\rho\rho P} = G_{\rho\rho f} = 4G_{\rho\rho\rho} \,. \tag{25}$$

This result implies that the $\rho + A_2$ contribution is proportional to $M^2 + 1.25\,M$, a result which is consistent with the parametrization for $\sigma_{tot}(\pi^- p)$ given previously. Therefore, we write the $\rho + A_2$ contribution as

$$Dg_{\lambda'_N,\lambda'_\Delta;\lambda_N,\lambda_\Delta} = G_\rho \, e^{A_\rho t} V_{\lambda'_\Delta\lambda'_N} V_{\lambda_\Delta\lambda_N} \left(\frac{s}{M^2}\right)^{2\alpha_\rho(t)} \Gamma(1 - \alpha_\rho(t))^2$$

$$\times M^2 \sigma_{tot}(\pi^- p)\,(p_\perp)^{|n|+|n'|} \,, \tag{26}$$

where $n = m$ for $m \neq 0$ and $= 2$ for $m = 0$ and similarly for n' in terms of m'. The ρ–A_2 trajectory is chosen to be $\alpha_\rho(t) = 0.5 + 0.9\,t$. The $\rho + A_2$ contribution thus involves two parameters, G_ρ and A_ρ. Eqs. (24) and (26) may be used to compare the model with data. There are, at this point, four parameters: $G^2_{\pi N\Delta}/4\pi$, A_π, G_ρ and A_ρ. In principle, the $\pi N\Delta$ coupling constant can be fixed at the value given earlier.

The data to be used in this analysis are at 205 GeV/c [27] and are presented in the form of $d\sigma/dx$ for $p_\perp^2 \leqslant 0.1$ (GeV/c)^2 and $d\sigma/dp_\perp^2$. * In each case $|t_{p\Delta}| \leqslant 1$ (GeV/c)^2. Therefore, the expressions given above must be integrated over the appropriate kinematic region in order to compare with the data. Furthermore, the data correspond to a selection $1.12 \leqslant m_{p\pi^+} \leqslant 1.32$ GeV/c^2 and the model must be corrected for this. Inserting the appropriate p-wave Breit–Wigner into the theoretical expressions and integrating over the restricted $p\pi^+$ mass range given above yields a correction factor of 0.65 which must be applied to the model. Finally, the distributions given in ref. [27] have not been corrected for the background under the Δ. The distributions as presented correspond to a cross section of 2 mb whereas the background subtracted cross section (for the $p\pi^+$ mass region quoted above and for $|t_{p\Delta}| \leqslant 1$ (GeV/c)^2) is 1.3 ± 0.14 mb. Therefore, the distributions must be scaled down by a factor of 0.65 in order to achieve the correct normalization. Notice that both of the correction factors cancel so that our conclusions are not sensitive to their precise values.

* Note that the data are presented in the region $x < 0$ corresponding to a slow $p\pi^+$ system in the lab frame. Of course, from the standpoint of the model, there is no difference between the $x < 0$ and $x > 0$ regions.

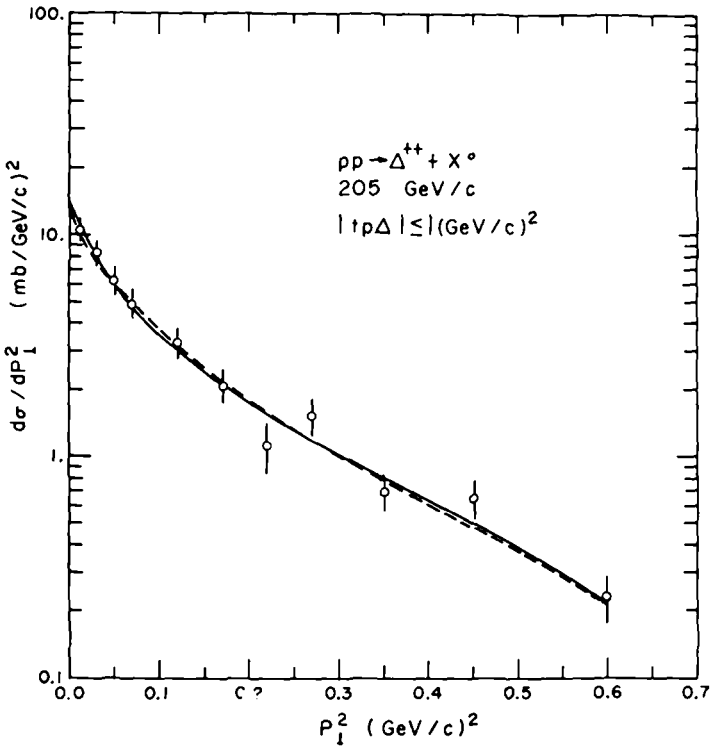

Fig. 1. The pole model (dashed line) and absorption model (solid line) fits to $d\sigma/dp_\perp^2$.

The simple pole model is compared with the data in figs. 1 and 2 where the fit is shown by the dashed line. The parameter values are given in table 2. The model provides an excellent description of the data, but this is done by reducing the value of the $\pi N\Delta$ coupling constant substantially. This observation was made in ref. [27] as well. This is somewhat disturbing since the π exchange Born term is known to give

Table 2
The parameter values for the two models (the underlined parameter were fixed prior to the fit)

Parameters	Poles only	Absorption model
$G^2_{\pi N\Delta}/4\pi$	9.0 GeV^{-2}	17.4 GeV^{-2}
A_π	1.5 (GeV/c)$^{-2}$	1.1 (GeV/c)$^{-2}$
G_ρ	19.3 GeV^{-6}	62.8 GeV^{-6}
A_ρ	-4.0 (GeV/c)$^{-2}$	-4.0 (GeV/c)$^{-2}$
C		1.02
a		5.94

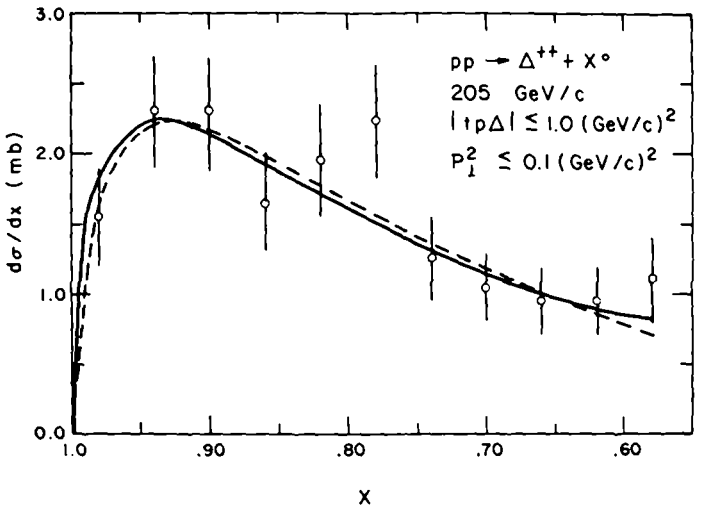

Fig. 2. The fits to dσ/dx. The curves are as in fig. 1.

a good description of the cross section for $\gamma N \to \pi^{\pm}\Delta$ (see for example ref. [28]). In ref. [27] agreement with the data in fig. 2 was obtained by using a value of $A_{\pi} = 6$ GeV^{-2}. This effectively reduces the π exchange contribution allowing the known value of the $\pi N\Delta$ coupling constant to be used. However, this procedure does not allow the $p_{\perp}^2$ distribution to be described simultaneously.

A second method of obtaining a good description of the data, while keeping the $\pi N\Delta$ coupling constant fixed, is to allow for the presence of absorptive corrections. The absorption has been calculated using the simple Gaussian parametrization given in eq. (16). The details of the relevant expressions are presented in the appendix. The absorption model results are shown in figs. 1 and 2 by the solid lines and the parameter values are given in table 2. Again, an excellent description of the data is obtained, this time with the $\pi N\Delta$ coupling fixed. The value for the absorption parameter a was estimated from the slope of elastic pp scattering. The parameter C was then varied while holding the $\pi N\Delta$ coupling constant fixed. The result in table 2 shows that the S wave ($b = 0$) is completely absorbed, similar to what is found in two-body reactions.

In fig. 3 the invariant cross section is shown, for both models, as a function of $p_{\perp}^2$ for fixed x. The absorption model result is also shown separately in fig. 4 to emphasize the x dependence. Clearly, the absorption model introduces no new structure, but rather simply reduces the cross section by an overall scale factor. This feature will be discussed in more detail in the next section. Several characteristic features of both models are evident in figs. 3 and 4. At $x = 0.95$ the π exchange peak is clearly evident – this comes from the non-flip, non-evasive amplitude $Dg_{1/2,1/2;1/2,1/2}$ As x decreases, M^2, and hence t_{min}, increases thereby reducing the effect of the π pole. Furthermore, the characteristic effect of Regge shrinkage can be seen with the

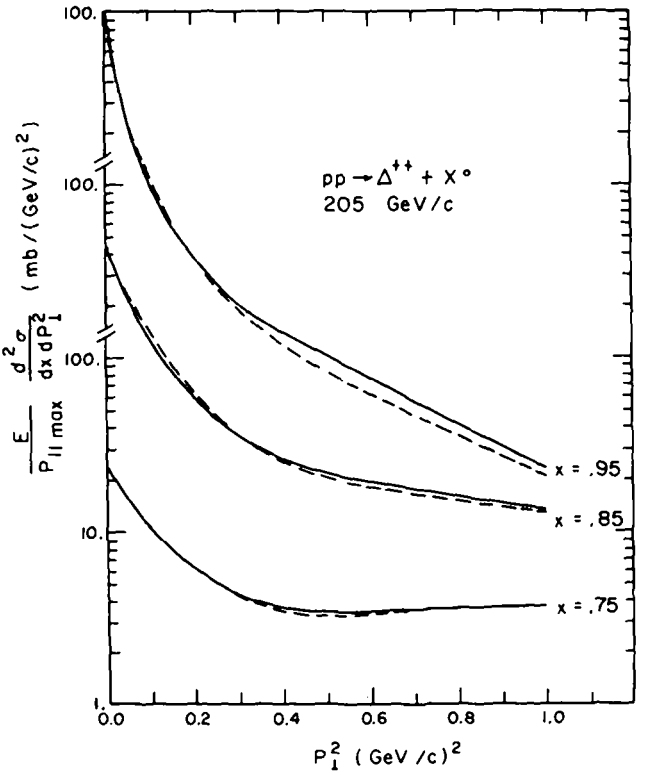

Fig. 3. Predictions of the two models for $(E/p_{\|\max})\,(\mathrm{d}^2\sigma/\mathrm{d}x\,\mathrm{d}p_\perp^2)$.

slope decreasing as x decreases. At the smaller x values for $p_\perp^2 \gtrsim 0.3$ (GeV/c)^2 the ρ and A_2 terms clearly dominate giving a very shallow $p_\perp^2$ dependence. However, the precise shape of the cross section in this region is difficult to determine from the data in figs. 1 and 2 due to the kinematic cut-off $|t_{p\Delta}| \leqslant 1$ (GeV/c)^2 *. In fig. 1, for example, only the two data points at $p_\perp^2 = 0.45$ and 0.60 (GeV/c)^2 are sensitive to the ρ and A_2 contributions. Therefore, the parameter values given in table 2 for the ρ and A_2 are only approximate. In addition, the Stodolsky-Sakurai coupling causes the $\rho + A_2$ terms to vanish at $p_\perp^2 = 0$, thereby making it even harder to determine the precise form of these terms.

In figs. 5–7 the predictions for ρ_{33}, $\mathrm{Re}\,\rho_{31}$, and $\mathrm{Re}\,\rho_{3-1}$ are given in the s channel helicity frame for both models as functions of $p_\perp^2$ at fixed x. Again it can be seen that no significant structure is introduced by the absorptive corrections. The structure of the density matrix elements is easy to understand. For small $p_\perp^2\pi$ ex-

* This cut off means that $p_\perp^2 < 1$ (GeV/c)^2 at $x = 1$ and $p_\perp^2 = 0$ at $x \approx 0.52$. Thus, only the data near $x = 1$ have $p_\perp^2$ values in the region where the ρ and A_2 terms dominate.

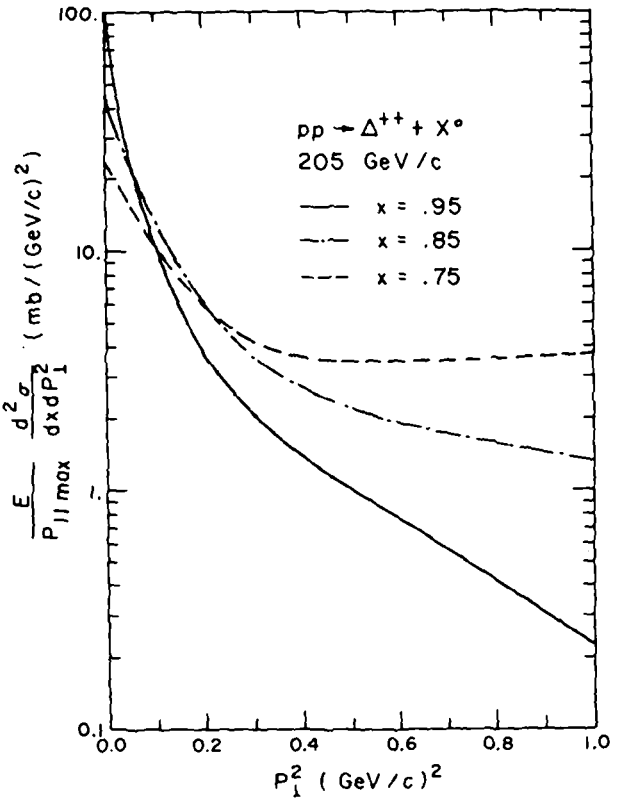

Fig. 4. The absorption model curves for $(E/p_{\parallel \max})\,(\mathrm{d}^2\sigma/\mathrm{d}x\,\mathrm{d}p_{\parallel}^2)$.

change dominates with $Dg_{1/2,1/2;1/2,1/2}$ being the largest amplitude. This suppresses ρ_{33} and enhances $\mathrm{Re}\,\rho_{31}$ and $\mathrm{Re}\,\rho_{3-1}$. At larger $p_\perp^2$ values the density matrix elements tend to the usual Stodolsky-Sakurai values of $\rho_{33} = \frac{3}{8}$, $\mathrm{Re}\,\rho_{31} = 0$, and $\mathrm{Re}\,\rho_{3-1} = \sqrt{\frac{3}{8}}$. It is interesting to note that ρ_{33} is not required to vanish in the forward direction, as may easily be seen using the results obtained in ref. [16]. Indeed, the absorptive corrections also yield a non-zero result at $p_\perp^2 = 0$, but it is so small that it cannot be seen in fig. 5. This result will be discussed in somewhat greater detail in sect. 4.

4. Characteristics of absorption

In fig. 8 the absorption model results for $Dg_{1/2,3/2;1/2,3/2}$ are shown for $x = 0.95$ and 0.75. The π pole term is required to vanish at $p_\perp^2 = 0$ as this amplitude has one unit of helicity flip at each $\pi N\Delta$ vertex. However, this amplitude is not required to

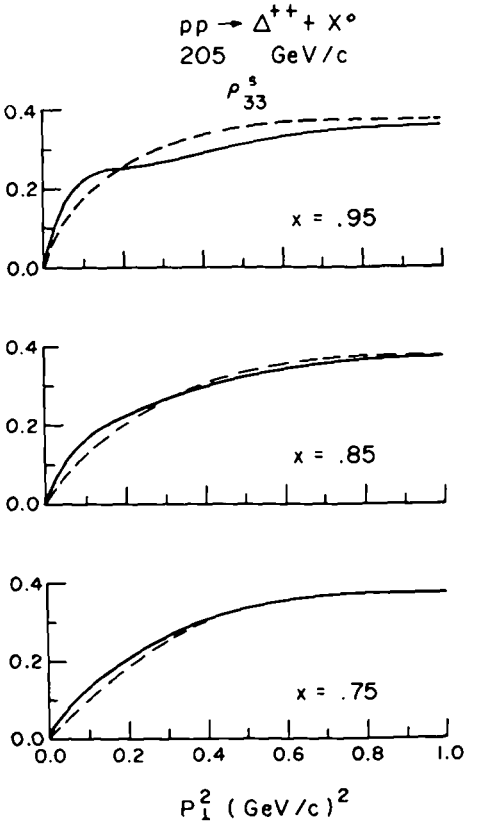

Fig. 5. The predictions for ρ_{33} in the s-channel helicity frame. The curves are as in fig. 1.

vanish at $p_\perp^2 = 0$ by angular momentum conservation. It is thus the analogue of the familiar evasive π exchange amplitude encountered in $\gamma N \to \pi^+ N$ for example. One might expect therefore, by analogy with two-body reactions, that absorption would yield a forward peak rather than a dip [29]. Fig. 8 shows that this is not the case, however. At $p_\perp^2 = 0$ the absorption is nowhere near strong enough to remove the dip. This effect may easily be understood in the following way [9]. The missing mass, M, produced in the inclusive reaction corresponds to a sum over all allowed helicities. Therefore, the inclusive amplitude is equivalent to a sum over the squares of quasi two-body helicity amplitudes with all values of the net helicity flip n. Now, only the $n = 0$ component of this sum is allowed to survive at $p_\perp^2 = 0$ and this is only a small part of the entire sum. The $n = 1, 2, \ldots$ components add to this, thereby washing out the effect when $p_\perp^2 > 0$.

This result is actually familiar from experience with separate two-body channels. Recall that in $\gamma N \to \pi^\pm N$ the π exchange contributes to only $n = 0$ and 2 amplitudes. Absorption is then capable of producing a forward peak. However, in $\gamma N \to \pi^\pm \Delta$

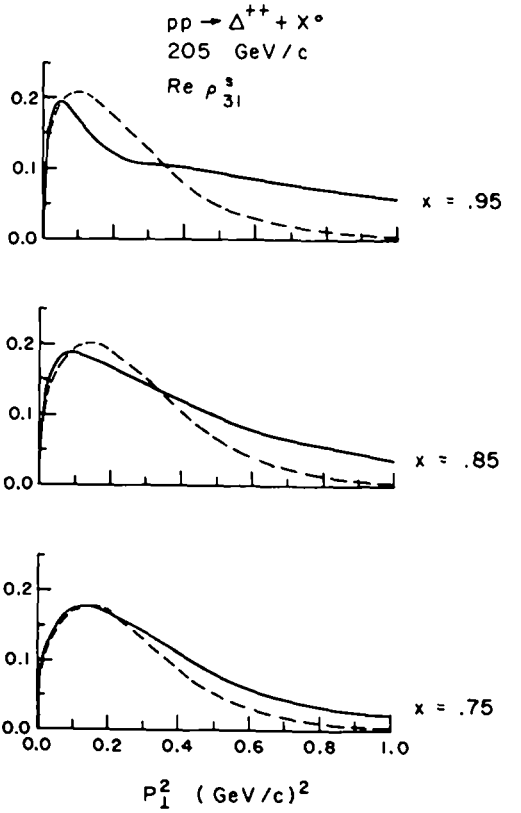

Fig. 6. The predictions for $\mathrm{Re}\rho_{31}$ as in fig. 5.

there is an $n = 1$ term as well which rises rapidly from 0 at $t = t_{min}$ to a maximum near $t = -m_\pi^2$. Thus, in Δ photoproduction a forward dip is observed [21]. These two channels form part of the inclusive reaction $\gamma N \to \pi^\pm X$ and it is easy to see that the absorption model then predicts a forward dip. Indeed, the amplitude in fig. 8 has the same structure as that occurring in $\gamma N \to \pi^\pm X$ apart from a different x dependence near $p_\perp^2 = 0$ which is due to the smaller t_{min} effects.

The above discussion indicates that the major effect of absorption in the triple-Regge region will be to reduce the cross section without introducing any significant structure. This makes it rather difficult to find reactions which can provide adequate tests of the absorption model. However, it is possible that this very lack of structure can be used to provide such a test. Consider the reaction $\pi^- p \to \pi^0 X$. The only exchange coupling at the $\pi^-\pi^0$ vertex is ρ exchange so this is a very clean reaction to work with. Now, suppose that a structureless pole term (i.e. one possessing no NWSZ's) is used as input to the absorption model. Then, the absorbed model will again have no significant dip structure [30]. However, a dual B6 model calculation

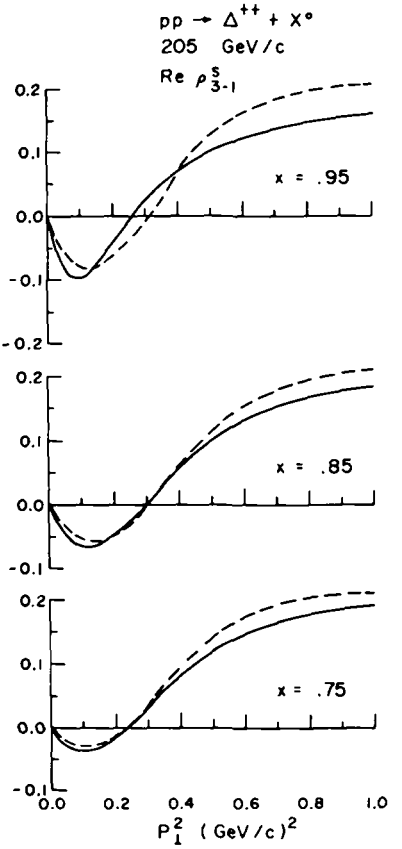

Fig. 7. The predictions for $\mathrm{Re}\rho_{3-1}$ as in fig. 5.

[31] indicates that the ρ triple-Regge term will indeed possess NWSZ's. The absorptive corrections will then convert the usual zero into a dip as in $\pi^- p \to \pi^0 n$ [32]. The important point is that the two models now predict strikingly different cross sections which is not the case in two-body reactions. This difference then allows a test of the absorption formalism. This is not a decisive test however, since a structureless cross section need not indicate absorptive effects. Also, a dip, if present, may be due to a ρ NWSZ plus a small background contribution which need not necessarily be due to absorption.

A more sensitive test of the absorption model would be one based on the phases of the amplitudes. For example, polarization effects in reactions where a simple Regge pole model would predict zero polarization could signal the presence of absorption. Also the analogue of line reversal breaking can be studied. For example, exchange degenerate ρ and A_2 Regge poles would predict that $K^+ p \to K^0 X$ and

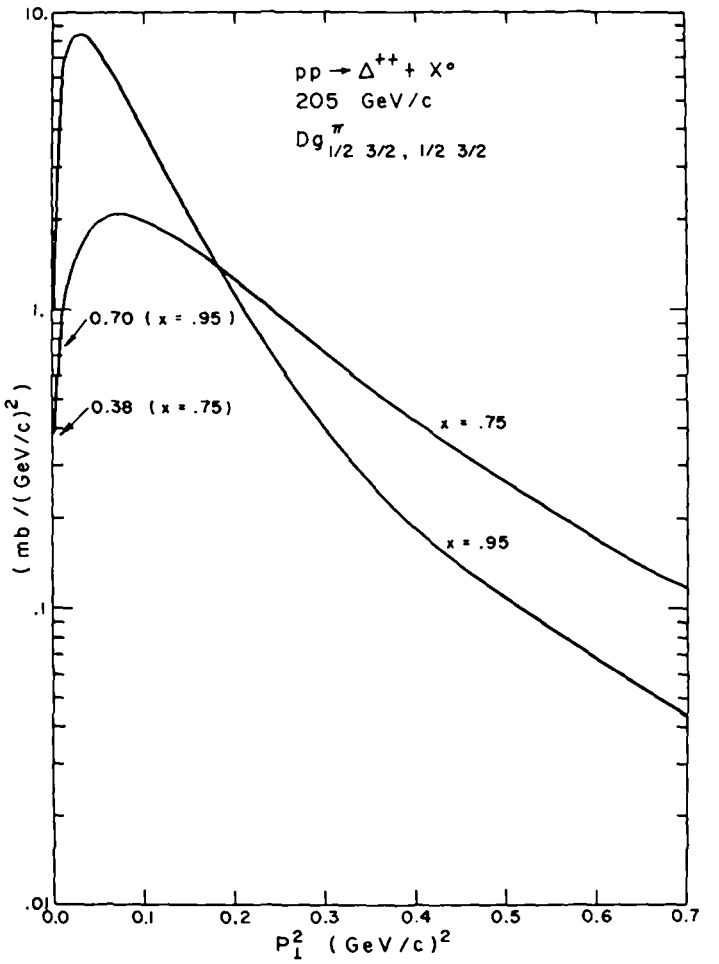

Fig. 8. The absorbed π exchange amplitude $Dg_{1/2\ 3/2,\ 1/2\ 3/2}$.

$K^- p \to \bar{K}^0 X$ should be equal in the triple-Regge region. An analysis of these reactions is currently in progress [32].

5. Conclusions

A comparison of the various approaches to absorptive corrections in the triple-Regge region has been given. The formalism has been applied to the reaction $pp \to \Delta^{++} X$ and an excellent description of the existing data has thereby been ob-

tained. In particular, the inclusion of absorption allows the $\pi N\Delta$ coupling constant to be fixed at its known value.

The absorptive corrections as calculated here do not add any significant structure not already present in the input pole terms. The major effect of the absorption is simply to reduce the magnitude of the resulting cross section. This makes it somewhat difficult to design decisive tests for the absorption model. However, effects which depend on the phases of the amplitudes can provide such tests. Thus, polarized target asymmetries in the triple-Regge region could be useful for such tests. Also, the analogue of line reversal breaking can be used to study the effects of absorption.

It is clear from the above discussion that inclusive reactions in the triple-Regge region can be used to study the effects of absorption at very high energies. The absorption model presented here is based on two-body phenomenology and, therefore, such a study is complementary to the usual two-body analyses. This is of particular interest since at Fermilab and ISR energies the inclusive cross sections are large enough for high statistics data to be obtained, whereas the various two-body reaction cross sections have fallen precipitously. The synthesis of the two types of analyses should therefore lead to a better understanding of the modifications to basic Regge pole exchange which must be made in order to describe the data.

One of the authors (JFO) wishes to thank Professor J. Pumplin for several interesting conversations.

Appendix

In this section the various expressions for the absorption model calculation presented in sect. 3 are presented. The starting point is the expression for the absorbed

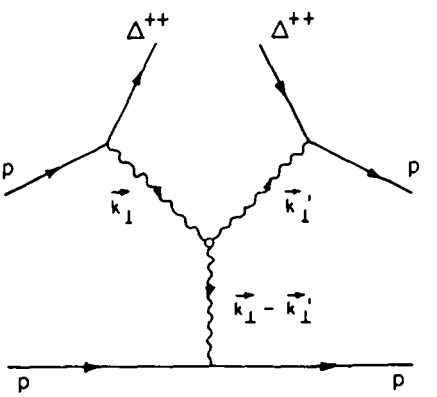

Fig. 9. The triple-Regge diagram and notation used in this analysis.

amplitude given in eq. (16) together with the input pole term expressions in eqs. (24) and (26). The first step is to write expressions for the non-forward triple-Regge pole terms. The notation is illustrated in fig. 9.

The transverse momenta are two-dimensional vectors defined as

$$\boldsymbol{k}_\perp = k_\perp(\cos\phi, \sin\phi) , \qquad \boldsymbol{k}'_\perp = k'_\perp(\cos\phi', \sin\phi') . \tag{A.1}$$

The upper vertices give rise to ϕ and ϕ' dependences of the form

$$\exp\{i(\lambda_N - \lambda_\Delta)\phi\} \exp\{-i(\lambda'_N - \lambda'_\Delta)\phi'\} .$$

Furthermore, the imaginary part of the reggeon-proton scattering amplitude must now be evaluated at a momentum transfer $(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)$. This momentum transfer dependence is parametrized as

$$\sigma(\pi^- p) = \beta_p(0)(M^2)^\alpha P^{(0)} \exp\{-B_p(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)^2\} + \beta_M(0)(M^2)^\alpha M^{(0)}$$
$$\times \exp\{-B_M(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)^2\} . \tag{A.2}$$

Here the subscript M denotes the sum of the ρ and f terms so that

$$\beta_p(0) = 21 \text{ mb} , \qquad \beta_M(0) = 26 \text{ mb} . \tag{A.3}$$

The parameter B_p can be estimated from elastic $\pi^- p$ scattering with the result

$$B_p = 0.3 \ln M^2 + 3.5 . \tag{A.4}$$

The f-coupled pomeron analysis in [25] suggests that

$$B_M = 0.9 \ln M^2 + 1.5 . \tag{A.5}$$

The absorption profile has been parametrized as in eq. (14) with $a = 0.3 \ln s + 4.15$ by comparison with the pp elastic scattering slope. Actually, a should contain $\ln s - \frac{1}{2} i\pi$. However, this small imaginary part will have a negligible effect on the results of this analysis and it has therefore been ignored. Similarly, the parameter C has been taken to be real. For analyses such as those discussed in sect. 4, where amplitude phases are important, a more sophisticated absorption profile must be used.

The above equations specify the necessary non-forward behavior of the pole terms so that the integrations in eq. (16) can now be performed numerically. However, it is possible to make some simple numerical approximations so that all of the integrations may be done analytically. Specifically, we use

$$\Gamma(-\alpha_\pi(t)) \approx \sum_{i=1}^{6} A_i e^{B_i t} ,$$

with

$$A_1 = 16.6411\,, \qquad B_1 = 89.4551\,,$$

$$A_2 = 16.1474\,, \qquad B_2 = 46.6987\,,$$

$$A_3 = 14.1941\,, \qquad B_3 = 25.2316\,,$$

$$A_4 = 11.2353\,, \qquad B_4 = 12.3311\,,$$

$$A_5 = \ 7.51176\,, \qquad B_5 = \ 4.54897\,,$$

$$A_6 = 2.09123\,, \qquad B_6 = \ 0.539168\,, \tag{A.6}$$

and $\Gamma(1-\alpha_\rho(t)) \approx \mathrm{e}^{h\alpha_\rho(t)}$ with $h = 1.145$.

The absorbed π + B helicity amplitudes can now be written in the general form

$$Dg_{\lambda'_N\lambda'_\Delta,\,\lambda_N\lambda_\Delta} = \beta^\pi_{\lambda'_\Delta\lambda'_N}\beta^\pi_{\lambda_\Delta\lambda_N}\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t_{\min})}\mathrm{e}^{A_\pi(t_{\min}-m_\pi^2)}$$

$$\times \sum_{k=\mathrm{P,M}} \beta_k(0)\,(M^2)^{\alpha_k(0)}\{(p_\perp)^{|m|+|m'|}\Gamma(-\alpha_\pi(t))^2\,\mathrm{e}^{-2\,\mathrm{Re}\,B_\pi p_\perp^2}$$

$$-\Gamma(-\alpha_\pi(t))\sum_{i=1}^{6} A_i\,\mathrm{e}^{B_i t_{\min}}[Ca(p_\perp)^{|m'|}I^2_{ik}(m)\,\mathrm{e}^{-B^*_\pi p_\perp^2} + C^*a^*(p_\perp)^{|m|}I^3_{ik}(m')\,\mathrm{e}^{-B_\pi p_\perp^2}]$$

$$+|C|^2|a|^2\sum_{i=1}^{6}\sum_{i=1}^{6} A_iA_j\,\mathrm{e}^{(B_i+B_j)t_{\min}}\,I^4_{ijk}(m',m)\}\,. \tag{A.7}$$

The various functions appearing in eq. (A.7) are given by

$$I^2_{ik}(m) = \frac{(p_\perp)^{|m|}(B_k+a)^{|m|}}{(B_{\pi_i}+B_k+a)^{|m|+1}}\,\exp\{-p_\perp^2 B_{\pi_i}(B_k+a)/(B_{\pi_i}+B_k+a)\}\,,$$

$$I^3_{ik}(m) = I^2_{ik}(m)^*\,,$$

$$I^4_{ijk}(0,0) = (\mathrm{e}^{-p_\perp^2 R_{ijk}/D_{ijk}})/D_{ijk}\,,$$

$$I^4_{ijk}(1,1) = I^4_{ijk}(0,0)[B_k + p_\perp^2 N_{ik}N^*_{jk}/D_{ijk}]/D_{ijk}\,,$$

$$I^4_{ijk}(2,2) = I^4_{ijk}(0,0)[2B_k^2 + 4B_k p_\perp^2 N_{ik}N^*_{jk}/D_{ijk} + p_\perp^4(N_{ik}N^*_{jk})^2/D^2_{ijk}]/D^2_{ijk}\,,$$

$$I^4_{ijk}(0,-1) = I^4_{ijk}(0,0)\,p_\perp N^*_{jk}/D_{ijk}\,,$$

$$I^4_{ijk}(0,-2) = I^4_{ijk}(0,-1)\,p_\perp N^*_{jk}/D_{ijk}\,,$$

$$I^4_{ijk}(1,-1) = I^4_{ijk}(0,0)\, p_\perp^2 N_{ik} N^*_{jk}/D^2_{ijk} ,$$

$$I^4_{ijk}(-1,-2) = I^4_{ijk}(0,0)\, p_\perp N^*_{jk} [2B_k + p_\perp^2 N_{ik} N^*_{jk}/D_{ijk}]/D^2_{ijk} , \tag{A.8}$$

where

$$B_{\pi_i} = [\alpha'_\pi \ln s/M^2 + \tfrac{1}{2}A_\pi + B_i]/x ,$$

$$R_{ijk} = 2\,\mathrm{Re}\, a[(B_{\pi_i} + B^*_{\pi_j})B_k + B_{\pi_i} B^*_{\pi_j}] + |a|^2 (B_{\pi_i} + B^*_{\pi_j}) ,$$

$$D_{ijk} = (B_{\pi_i} + a + B_k)(B_{\pi_j} + a + B_k)^* - B_k^2 ,$$

$$N_{ik} = a^*(B_{\pi_i} + a + B_k) + aB_k ,$$

$$\beta^\pi_{\lambda_\Delta \lambda_N} = G_{\pi N\Delta} \alpha'_\pi V_{\lambda_\Delta \lambda_N} ,$$

$$\alpha_P(0) = 1 , \qquad \alpha_M(0) = 0.5 . \tag{A.9}$$

The absorbed $\rho + A_2$ terms can be written in a similar form.

$$Dg_{\lambda'_N \lambda'_\Delta, \lambda_N \lambda_\Delta} = \beta^\rho_{\lambda'_\Delta \lambda'_N} \beta^\rho_{\lambda_\Delta \lambda_N} \left(\frac{s}{M^2}\right)^{2\alpha_\rho(t_{\min})} e^{A_\rho t_{\min}} e^{2\alpha_\rho(t_{\min})h}$$

$$\times \sum_{k=P,M} \beta_k(0)\,(M^2)^{\alpha_k(0)} \{(p_\perp)^{|m|+|m'|} e^{-2\,\mathrm{Re}\, B_\rho p_\perp^2} - [Ca(p_\perp)^{|m'|} I^2_k(m)$$

$$+ C^* a^* (p_\perp)^{|m|} I^3_k(m')] + |C|^2 |a|^2 I^4_k(m', m)\} , \tag{A.10}$$

$$I^2_k(m) = \frac{(p_\perp)^{|m|}(B_k + a)^{|m|}}{(B_k + a + B_\rho)^{|m|+1}} \exp\left\{-p_\perp^2 \left[B^*_\rho + \frac{B_\rho (B_k + a)}{B_\rho + B_k + a}\right]\right\} \qquad m \neq 0 ,$$

$$I^2_k(0) = \frac{\exp\left\{-p_\perp^2 \left[B^*_\rho + \dfrac{B_\rho (B_k + a)}{B_\rho + B_k + a}\right]\right\}}{(B_k + a + B_\rho)^2} \left[1 + p_\perp^2 \frac{(B_k + a)^2}{B_k + a + B_\rho}\right] ,$$

$$I^3_k(m) = I^2_k(m)^* . \tag{A.11}$$

Let $H_k = [\exp\{-p_\perp^2 R_k/D_k\}]/D_k$. Then

$$I^4_k(0,0) = \frac{H_k}{D_k}\left\{1 + \frac{2B_k^2}{D_k} + \frac{p_\perp^2}{D_k}\left[\frac{|N_k|^2 - |a|^2 D_k}{B_k} + \frac{4B_k |N_k|^2}{D_k}\right] + \frac{p_\perp^4 |N_k|^4}{D_k 3}\right\} ,$$

$$I^4_k(1,1) = \frac{H_k}{D_k}\left[B_k + \frac{p_\perp^2 |N_k|^2}{D_k}\right] ,$$

$$I_k^4(2,2) = \frac{H_k}{D_k^2}\left[2B_k^2 + \frac{4p_\perp^2 B_k |N_k|^2}{D_k} + \frac{p_\perp^4 |N_k|^4}{D_k^2}\right],$$

$$I_k^4(0,-1) = \frac{H_k}{D_k^2} p_\perp \left[2N_k^*(B_\rho + a + B_k) - aD_k + p_\perp^2 \frac{|N_k|^2 N_k}{D_k}\right],$$

$$I_k^4(0,-2) = \frac{H_k p_\perp^2}{D_k^3}\left\{N_k^*[3N_k(B_\rho + a + B_k) - 2aD_k] + p_\perp^2 \frac{|N_k|^4}{D_k}\right\},$$

$$I_k^4(1,-1) = H_k p_\perp^2 |N_k|^2 / D_k^2 ,$$

$$I_k^4(-1,-2) = H_k p_\perp [2B_k + p_\perp^2 |N_k|^2 / D_k] N_k^* / D_k^2 .$$

In the above expressions

$$B_\rho = [\alpha'_\rho(\ln s/M^2 + h) + \tfrac{1}{2}A_\rho]/x , \qquad D_k = |B_\rho + a + B_k|^2 - B_k^2 ,$$

$$R_k = 2\,\mathrm{Re}\, a[|B_\rho|^2 + 2\,\mathrm{Re}\, B_\rho B_k] + 2\,\mathrm{Re}\, B_\rho |a|^2 ,$$

$$N_k = a^*(B_\rho + a + B_k) + aB_k , \qquad B^\rho_{\lambda_\Delta \lambda_N} = \sqrt{G_\rho} V_{\lambda_\Delta \lambda_N} . \tag{A.12}$$

The expressions appearing above have been written in such a way that the absorption and Regge-pole terms could be complex even though they are all real in this analysis. Thus, the expressions contained in this appendix are sufficiently general that more sophisticated absorption mechanisms could be chosen or exchange degeneracy breaking effects could be studied.

Note added in proof

After the completion of this work we received notice of an absorption model calculation for the reactions $(\pi^+, K^-)p \to \Delta^{++} + X$ [34]. This paper also stresses the need for including absorptive corrections in the triple-Regge region.

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